

Fermionic systems with charge correlations on the Bethe lattice

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Abstract

A fermionic model, built up of q species of localized Fermi particles, interacting by charge correlations, is isomorphic to a spin- $\frac{q}{2}$ Ising model. However, the equivalence is only formal and the two systems exhibit a different physical behavior. By considering a Bethe lattice with $q = 1$, we have exactly solved the models. There exists a critical temperature below which there is a spontaneous breakdown of the particle-hole symmetry for the first model, and of the spin symmetry for the second. While the spin system is always stable and exhibits a homogeneous ferromagnetic phase below T_c , the fermionic system for $T < T_c$ is unstable against the formation of inhomogeneous phases with charge separation.

Key words: Fermi systems, Bethe lattice, Ising model

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It is known [1] that there is an isomorphism between fermionic models, built up of q species of localized Fermi particles, interacting by charge correlations, and spin- $\frac{q}{2}$ Ising-like models. The fermionic system is described by the Hamiltonian

$$H_{ferm} = -\mu \sum_i n(i) + \frac{1}{2} z V \sum_i n(i) n^\alpha(i) \quad (1)$$

where $n(i) = \sum_{a=1}^q c_a^\dagger(i) c_a(i)$ is the total particle density, $c_a(i)$ and $c_a^\dagger(i)$ being the annihilation and creation operators of the species a in the Heisenberg picture; $i = (\mathbf{i}, t)$, where \mathbf{i} stands for the lattice vector \mathbf{R}_i . These operators satisfy canonical anti-commutation relations. z is the coordination number of the underlying lattice, V is the strength of the intersite interaction and μ is the chemical potential. The spin system is described by the Hamiltonian

$$H_{spin} = -h \sum_i S(i) - \frac{1}{2} z J \sum_i S(i) S^\alpha(i) \quad (2)$$

where $S(i)$, the spin operator at the site \mathbf{i} , takes the $q + 1$ values $-\frac{q}{2}, \dots, \frac{q}{2}$; J is the exchange interaction and h is the external magnetic field. We are considering systems with first-nearest neighbor interactions; for a generic operator $\Phi(i)$ we use the notation $\Phi^\alpha(i) = \sum_j \alpha_{ij} \Phi(j, t)$, where α_{ij} is the projector on the first-nearest neighbor sites. The equivalence of the two models, $H_{ferm} = E_0 + H_{spin}$, is based on the following relations

$$\begin{aligned} n(i) &= \frac{q}{2} + S(i) & V &= -J \\ \mu &= h + \frac{q}{2} z V & E_0 &= -\frac{q}{2} \left(\mu - \frac{q}{4} z V \right) N \end{aligned} \quad (3)$$

where N is the number of sites. The relation between the partition functions is $Z_{ferm} = e^{-\beta E_0} Z_{spin}$, and the thermal average of any operator A assumes the same value on both models $\langle A \rangle_{ferm} = \langle A \rangle_{spin}$. We have shown [1] that these systems are exactly solvable. This means that it is always possible to find a complete set of eigenoperators and eigenvalues of the Hamiltonian (1) and/or (2) which close the hierarchy of the equations of motion. In such a way,

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formal exact expressions for the relevant Green's functions and correlation functions can be derived, which depend on a finite set of parameters to be self-consistently determined. It has been shown how to fix exactly such parameters by means of algebra constraints [2] in the case of a linear chain and $q = 1, 2, 3$ [1], and in the case of the Bethe lattice with any z and $q = 1$ [3]. However, the equivalence of the two models is just formal. There is an enormous difference from a physical point of view. The reason is the following. For the spin system, the external thermodynamical parameters are h and T : the system responds to these parameters by a certain configuration of the spin, described by the magnetization $m = \langle S \rangle$. For the fermionic model, the external thermodynamical parameters are n and T , where n is the particle density: the system responds to these parameters by adjusting the chemical potential μ . In order to illustrate these differences, we have studied the two models (1) and (2) on the Bethe lattice with $z = 3, 4$, by considering the case $J > 0$ (i. e. ferromagnetic coupling). According to the exact solution given in Ref. [3], there is a critical temperature T_c such that for $T < T_c$ there is a spontaneous breakdown of the symmetry enjoyed by the two models: Hamiltonian (2) is invariant under the transformation $S \rightarrow -S, h \rightarrow -h$. In the fermionic system this transformation corresponds [cfr. (3)] to the particle-hole transformation $\mu \rightarrow -\mu + qzV, n \rightarrow -n + q$. At $n = \frac{q}{2}$, where $\mu = \frac{zqV}{2}$, the Hamiltonian (1) is invariant and enjoys the symmetry. For the spin system the critical temperature $T_c(h)$ and the magnetization m are shown in Figs. 1 and 2, respectively, for $z = 3$. For general z , $T_c(h)$ decreases from the value $k_B T_c = \frac{2J}{\log(\frac{z}{z-2})}$ at $h = 0$ and vanishes at $|h| = J(z-2)$. As seen in Fig. 2, for $T < T_c$ there is a spontaneous magnetization in zero field.

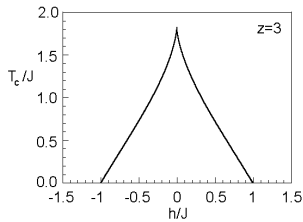


Fig. 1. The temperature $T_c(h)$ is plotted against the magnetic field h for $z = 3$.

For the fermionic system the physical situation is rather different. The critical temperature $T_c(n)$ and the chemical potential are shown in Figs. 3 and 4, respectively. For $T < T_c(n)$, in correspondence of a

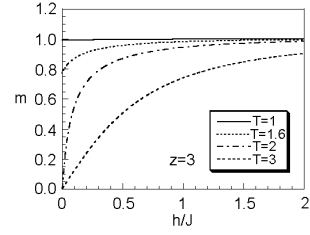


Fig. 2. The magnetization m is plotted against h/J for $z = 3$ and several values of the temperature.

fixed value of the chemical potential there are three solutions for the particle density. As clearly seen in Fig. 4, n_2 corresponds to an unstable solution (the compressibility is negative). In conclusion, while the spin system is always stable and exhibits a homogeneous ferromagnetic phase below T_c , the fermionic system for $T < T_c$ is unstable, except small regions around $n \approx 0$ and $n \approx 1$, against the formation of inhomogeneous phases with charge separation.

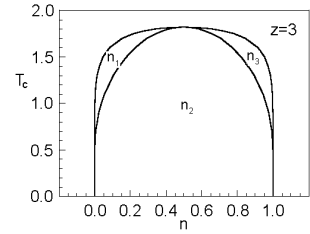


Fig. 3. The phase diagram in the space $T_c - n$ for $z = 3$.

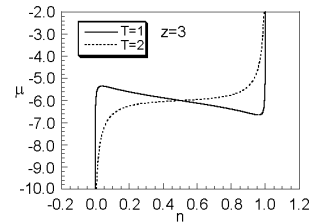


Fig. 4. The chemical potential μ is plotted versus n for $z = 3$.

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